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INFLUENCE OF THE SIZE FACTOR ON THE ACCURACY OF VOLT-AMPERE CHARACTERISTICS OF A VORTEX ELECTRIC-ARC PLASMATRON WITH TUBULAR ELECTRODES

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Statistical methods of analysis are used to study the influence of a change in the electrode diameter on the accuracy of the volt-ampere characteristics (VACs) of a vortex electric-arc plasmatron operating with technical nitrogen. It is shown that when each initial parameter is allowed for separately and the VACs are represented by an empirical formula, the influence of a change in the diameter is insignificant. When all the initial parameters are allowed for simultaneously in one dominant criterion in the case of approximation of VACs by a generalized expression, the effect of the size factor proves to be substantial.

In designing plasmatrons, it is necessary to know their characteristics that relate the initial controlled parameters and the output technological parameters. For electric-arc plasmatrons, the most important is the volt-ampere characteristic (VAC), which reflects the energy features of the apparatus, which converts electrical energy to heat energy, and describes its properties as an electric circuit load. Vortex-type electric-arc plasmatrons, in which the arc is blown by heated gas that insulates the discharge from the walls of a tubular electrode, are most frequent used for various purposes. This blowing does not ensure strict stabilization of the sizes of the arc column and its position along the axis of the cylindrical channel. Therefore, it is practically impossible to theoretically calculate VACs of discharges that are unstable in space and time. Accordingly, designers of plasmatrons must rely on physical modeling with the use of experimental data on the characteristics of electric-arc apparatuses.

To improve the physical modeling of electric-arc plants, methods of approximate similarity of blown arc discharges have been developed in which criterial expressions for generalized variables are used. Ordinary empirical formulas that express the relations between the determined parameters and the initial variables are also employed. The choice of one or another method is dependent on the required accuracy, the complexity of the processing of experimental data, and the versatility of the expressions obtained. At present, generalized formulas are usually preferred. Since an electric-arc discharge is a very complicated process that is dependent on a great number of factors, we must resort to approximate methods. Therefore, it is necessary to assess the degree of the influence of individual initial variables and generalized arguments on the function in question and to select the most important parameters.

The choice of important parameters is dependent on the accuracy of the expression obtained: an argument can be considered unessential if the degree of its influence on the function is found to be within the limits of the spread of experimental data. Towards this end, statistical methods of dispersion analysis are usually used [1]. However, as far as the characteristics of electric-arc discharges are concerned, the statistical approach has still not found extensive use. To somewhat make up for this gap, we shall attempt to assess the influence on vortex-plasmatron VACs of such an important factor as the electrode diameter. An evaluation is performed for a unilateral-emission plasmatron with two tubular electrodes that operates on technical nitrogen (Fig. 1). The approximation of VACs by both generalized and ordinary empirical formulas is considered.

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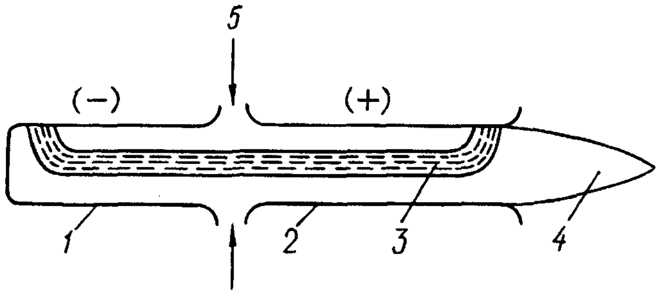

Fig. 1. Vortex plasmatron with two tubular electrodes: 1) cathode; 2) anode; 3) electric arc with length that is self-adjusting by shunting; 4) plasma jet; 5) gas supply.

The electrodes of the plasmatron had equal diameters of 10, 20 and 40 mm. The rear dead-end electrode served as the cathode, while the outlet flow cylinder served as the anode. The flow rates of the gas for each electrode size were 2.4 and 6 g/sec; the current varied from 40 to 900 A.

It was shown earlier [2] that simple power expressions of the form $U = cI^{\alpha}G^{\beta}d^{\gamma}$ (a simple empirical formula) or $(Ud/I) = c(Gd/I^2)^{\alpha}$ (a generalized criterial equation) are quite suitable for approximating VACs of electric-arc plasmatrons. These formulas are convenient for statistical processing, since, in a logarithmic form, they are a simple linear regression. Therefore, power expressions are employed in the present work.

In assessing the degree of influence for any of the parameters, we can use two approaches. The first approach implies that, for the same mass of experimental points, we compare two approximating expressions, one of which allows for the assessed parameter while the other does not. As applied to the consideration of the influence of electrode diameter, these empirical formulas are $U = cI^{\alpha}G^{\beta}d^{\gamma}$ and $U = cI^{\alpha}G^{\beta}$. In both cases, we calculate the sums of the squares of the deviations from the regression SS_{dev} and their corresponding numbers of degrees of freedom N_{dev}. It is evident that $SS_{\text{dev}(2)} > SS_{\text{dev}(3)}$ and $N_{\text{dev}\,(2)} - N_{\text{dev}\,(3)} = 1$, where the subscripts in parentheses indicate the number of the arguments in the formula. $SS_{\text{add}} = SS_{\text{dev}(2)} - SS_{\text{dev}(3)}$ is the increase in the sum of the squares of the deviations from the regression when the assessed factor is ignored. This sum corresponds to the number of the degrees of freedom $N_{\text{add}} = 1$. From the above data, we can determine the ratio of the Fisher variances

$$F_{\text{add}} = \frac{S^2_{\text{add}}}{S^2_{\text{dev}}} = \frac{SS_{\text{add}}}{N_{\text{add}}} \Big/ \frac{SS_{\text{dev}}}{N_{\text{dev}}}, \tag{1}$$

from which the influence of the given factor is assessed by comparing F_{add} with the tabulated value of the Fisher number F_{tab}. If $F_{\text{add}} < T_{\text{tab}}$ the influence of the factor in question does not fall outside the limits of random error and can be disregarded.

In generalized formulas, each argument reflects the influence of a process described by a complex of individual variables, rather than any specific factor. In vortex-type plasmatrons, convective heat transfer, which is expressed by the number $\pi_{\text{conv}} = Gd\sigma_0 h_0/I^2$, is the most important for VACs. For specific gases, use is usually made of a dimensional form of this number in which the characteristic properties of the plasma $\pi_{\text{conv}} = Gd/I^2$ are omitted. Therefore, the use of the considered method for assessing the role of individual arguments in the case of a generalized expression shows the influence of the processes in the discharge rather than that of the individual variables.

The influence of individual parameters can be allowed for by different approach, in which the stratification of the regression by a parameter is assessed. The Fisher number, in this case, is defined as the ratio of the inadequacy variance to the net error variance $F_{\text{inad}} = S^2_{\text{inad}}/S^2_{\text{n.err}}$. As applied to the case in question, the sum of the squares of the deviations in the expressions $U = cI^{\alpha} - G^{\beta}$, which are calculated for each diameter, can be taken as the sum of the squares of the deviations for the net error, i.e., $SS_{\text{n.err}} = \sum\limits_i SS_{\text{dev}(i)}$. If this quantity is subtracted from the sum of the squares of the deviations in the expression $U = cI^{\alpha}G^{\beta}$, which is calculated for all diameters together with $SS_{\text{dev}(0)}$,

TABLE 1. Parameters of Correlation of VAC of Vortex Plasmatron with Two Tubular Electrodes and Arc Blowing by Technical Nitrogen

Approximation formula	Root-mean-square deviation from regression Δ	Student quantiles			Correlation factors						Fisher correlation $F_{regr} = \frac{S^2_{regr}}{S^2_{dev}}$	F_{tab}	
		t_I	t_G	t_d	$U-I$	$U-G$	$U-d$	$I-G$	$I-d$	$G-d$		$\alpha = 5\%$	$\alpha = 1\%$
$U = cI^{\alpha}$	0.121	1.13	–	–	−0.137	–	–	–	–	–	1.28	3.14	4.94
$U = cG^{\beta}$	0.085	–	8.49	–	–	0.720	–	–	–	–	72.1	3.14	4.94
$U = cd^{\gamma}$	0.120	–	–	1.79	–	–	0.214	–	–	–	3.21	3.14	4.94
$U = cI^{\alpha}G^{\beta}$	0.034	−18.7	28.0	–	−0.137	0.720	–	0.549	–	–	399	2.75	4.10
$U = cI^{\alpha}d^{\gamma}$	0.119	−1.07	–	1.74	−0.137	–	0.214	–	−0.046	–	2.18	2.75	4.10
$U = cG^{\beta}d^{\gamma}$	0.082	–	8.73	2.41	–	0.720	0.214	–	–	0.023	41.5	2.75	4.10
$U = cI^{\alpha}G^{\beta}d^{\gamma}$	0.029	−2.18	32.9	5.33	−0.137	0.720	0.214	0.549	−0.046	0.023	387	2.51	3.62

TABLE 2. Determining the Appropriateness of Complication of an Empirical Formula by Allowing for the Influence of the Electrode Diameter

Parameters of regression	Value of parameter		
	$U = cI^{\alpha}G^{\beta}$	$U = cI^{\alpha}G^{\beta}d^{\gamma}$	estimate
Sum of the squares of deviation from regression SS_{dev}	0.07653	0.05324	0.02329
Number of degrees of freedom, N_{dev}	66	65	1
Variance of deviation, S^2_{dev}	$1.1596 \cdot 10^{-3}$	$8.1909 \cdot 10^{-4}$	$2.329 \cdot 10^{-2}$
Variance ratio for regression, F_{regr}	399	387	–
Variance ratio for additional term, F_{add}	–	–	20.08
Tabulated values of F (5%/1%)	2.75/4.10	2.51/362.0	3.99/7.09

we can find the sum of the squares of the deviations of the inadequacy, which, in the given case, is produced due to regression stratification by individual diameters: $SS_{\text{inad}} = SS_{\text{dev}(0)} - SS_{\text{n.err}} = SS_{\text{dev}(0)} - \sum_i SS_{\text{dev}(i)}$. Having determined in a similar manner the numbers of degrees of freedom $N_{\text{n.err}} = \sum_i N_{\text{dev}(i)}$, $N_{\text{inad}} = N_{\text{dev}(0)} - N_{\text{n.err}}$, we can determine the variables $S^2_{\text{n.err}} = \sum_i SS_{\text{dev}(i)} / \sum_i N_{\text{dev}(i)}$, $S^2_{\text{inad}} = SS_{\text{inad}}/N_{\text{inad}}$. If $F_{\text{inad}} = S^2_{\text{inad}}/S^2_{\text{n.err}}$ is smaller than the tabulated value of the Fisher ratio, the stratification of the generalized expression due to a change in the parameter in question (in our case, the electrode diameter) does not fall outside the limits of random error.

Certain conclusions regarding the degree of the influence of different parameters on plasmatron VACs can be drawn from the data of regression analysis. Table 1 gives the parameters of the regression for generalizing the experimental results of an investigation of a plasmatron with tubular electrodes when the arc was blown by technical nitrogen (69 experimental points). Approximation was performed by empirical power formulas. Variants of the dependence of the arc voltage on each individual parameter (I, G, d) and on their set were considered.

Table 1 shows that the dependence of the voltage on the current and diameter proves to be weak. The tabulated value of the Student quantile for a number of degrees of freedom larger than 30 and a probability of 5% is $t = 1.96$, while for a probability of 1% it increases to $t = 3.29$. For both the current and the diameter, the obtained values of the Student quantiles are lower than the above values. The same assessment is also obtained for the regression ratio of the variances $F_{\text{regr}} = S^2_{\text{regr}}/S^2_{\text{dev}}$. The correlations between the voltage and these parameters prove to be low, too.

The weak dependence of the voltage on the current is governed by the low degree of arc stabilization in vortex plasmatrons whose arc-column length is self-adjusting by shunting. Owing to this as the current grows the resistance of the arc decreases, both due to an increase in the diameter of the column and due to a reduction in its length. As a result, the VAC of a high-current arc proves to be almost parallel to the axis of the currents ($U \approx$ const). In the experiments in question, a deviation of the voltage from a constant value with changing current was found to be within the limits of error. Correspondingly, the calculated values of the Student quantiles and Fisher regression proved to be lower than the tabular ones. The dependence of the voltage on the electrode diameter proved to be more pronounced than on the current but also rather weak.

However the main reason for the low values of regression parameters for the correlation of a VAC in the form $U = cI^{\alpha}$ is the strong dependence of the voltage on the flow rate of the gas. If it is allowed for by an approximation of the form $U = cI^{\alpha}G^{\beta}$, the Student quantile for the current grows to $t_I = -18.7$, which is much higher than the tabulated value. Therefore, as the flow rate of the gas changes we cannot perform a correlation of VAC in the form $U = cI^{\alpha}$. When the dependence of the voltage on the gas flow rate in the form of the correlation $U = cG^{\beta}d^{\gamma}$ is allowed for, the influence of the electrode diameter becomes pronounced, too. In this case, t_d increases to 2.41, which is higher than the tabulated value. The value of t_d grows still further when the influences of both the

TABLE 3. Parameters of Regression for VAC of Vortex Plasmatron with Tubular Electrodes of Different Diameter with Changing Electrode Diameter

Electrode diameter, cm	Form of approximation	Exponents on arguments		Root-mean-square deviation from the regression, Δ	Correlation factors			Student quantiles		Variance ratio for the regression, F_{regr}
		α	β		f–arg.1	f–arg.2	arg.1–arg.2	t_α	t_β	
1	$U = cI^\alpha G^\beta$	−0.435	0.717	0.0258	−0.098	0.655	0.671	−14.5	19.4	190
2	–	−0.328	0.625	0.0193	−0.111	0.804	0.483	−15.5	26.7	360
4	–	−0.389	0.684	0.0324	−0.200	0.725	0.498	−12.2	18.0	169
Entire set	–	−0.391	0.684	0.0753	−0.137	0.720	0.549	−18.7	28.0	399
1	$Ud/I = c(Gd/I^2)^\alpha$	0.717	0.0251	0.0251	0.996	–	–	52.0	–	2702
2	–	0.665	–	0.0204	0.997	–	–	59.6	–	3550
4	–	0.695	–	0.0317	0.995	–	–	44.8	–	2009
Entire set	–	0.802	–	0.0829	0.978	–	–	38.5	–	1482

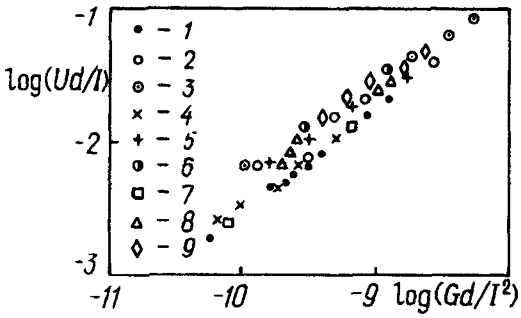

Fig. 2. Generalized VAC of vortex plasmatron. (The working gas is nitrogen): 1, 2, 3) $G = 2$ g/sec; 4, 5, 6) $G = 4$ g/sec; 7, 8, 9) $G = 6$ g/sec; 1, 4, 7) $d =$ 10 mm; 2, 5, 8) $d = 20$ mm; 3, 6, 9) $d = 40$ mm.

current and the gas flow rate are allowed for simultaneously. However, the change in the diameter has a rather slight effect, and in some cases, with the aim of simplifying the empirical formula, we can write it as $U = cI^{\alpha}G^{\beta}$. The values of F_{regr} and of the root-mean-square deviation change little.

Now let us see how well the assessment by the Student quantile agrees with that by the change in the deviation of the experimental points from the regression. We determine the ratio of the Fisher variances F_{add} using the more complex formula $U = cI^{\alpha}G^{\beta}d^{\gamma}$ instead of the expression $U = cI^{\alpha}G^{\beta}$. The calculation data for F_{add} for the same set of experimental points are given in Table 2.

The table shows that the influence of electrode size on the arc voltage falls outside the limits of random error, since $F_{\text{add}} > F_{\text{tab}}$. However, the absolute value of F_{add} is low as compared with F_{regr} (5%). The obtained result is in good agreement with the estimate by the Student quantiles.

We consider approximation by the generalized formula with the influence of the electrode size allowed for by the second method. For comparison, we also employ this method concurrently for the empirical formulas. Regression parameters for individual electrode diameters and the entire set as whole in an approximation as $U = cI^{\alpha}G^{\beta}$ and $Ud/I = c(Gd/I^2)^{\alpha}$ are given in Table 3.

In Table 3, for the correlation factors the letter f denotes a function that is equal to the voltage U or to the generalized resistance Ud/I in cases of approximation by simple or generalized formulas, respectively. Similarly, the current I or the complex (Gd/I^2) is represented by the argument arg. 1, while arg. 2 denotes the flow rate of the gas.

From the data of Table 3 it can be seen that there is a spread in the parameters as the eelctrode diameter changes. The indices for $d = 2$ cm were better (the smaller the root-mean-square deviation and, respectively, the larger the Student quantiles t_{α} and t_{β} and the ratio of the variances F_{regr}). Curiously, F_{regr} and the correlation factors improve substantially when the generalized formula is used as compared with the nongeneralized one. In the nongeneralized formula, a pronounced mutual correlation between the current and the gas flow rate is observed.

Calculation data on the parameters of curve stratification by the electrode diameter for both forms of approximation are tabulated (Table 4). In the case of the empirical formula, the obtained result is in good agreement with the previous assessments: the Fisher dissimilarity number exceeds the tabulated values $F_{\text{inad}} > F_{\text{tab}}$, but this excess is small. However, for the generalized formula F_{inad} proves to be rather significant, which indicates the substantial diameter stratification of generalized VACs of the form $(Ud/I) = c(Gd/I^2)^{\alpha}$. This is explained by the fact that such an important factor as the flow rate of the gas and the unimportant electrode diameter are brought together in a unified complex. The parameters of the regression are established under the action of the effective variable. Therefore, the spread of the unimportant quantity is increased. The stratification of the generalized VAC by diameter is noticeable even visually. Figure 2 shows the good convergence of the generalized VAC in the gas flow rate when one argument that accounts for convective heat transfer is used. This result is also confirmed by statistical analysis [3]. However, in this case we observed the stratification of the generalized VAC by electrode diameter.

It is pertinent to note that nongeneralized formulas of the form $U = cI^{\alpha}G^{\beta}d^{\gamma}$ contain three independent variables, while in the generalized expression, they are integrated into a combined argument. The fact that the

TABLE 4. Analysis of the Influence of Electrode Diameter on Stratification of VAC of Vortex Plasmatron

Parameters of regression	Values of parameters	
	$U = cI^{\alpha}G^{\beta}$	$Ud/I = c(Gd/I^2)^{\alpha}$
Sum of squares of deviations from regression for the total correlation of VAC for all electrode diameters, $SS_{dev(0)}$	0.07653	0.46079
Number of degrees of freedom, $N_{dev(0)}$	66	67
Sum total of squares of deviations for correlation of VAC for individual electrode diameters, $SS_{n.err}$	0.04139	0.04283
Number of degrees of freedom, $N_{n.err}$	60	63
Sum of squares of deviations due to VAC stratification by electrode diameters, SS_{inad}	$3.5149 \cdot 10^{-2}$	$4.1797 \cdot 10^{-1}$
Number of degrees of freedom, N_{inad}	6	4
Variance of deviations for individual electrode diameters, $S^2_{n.err}$	$6.8987 \cdot 10^{-4}$	$6.7979 \cdot 10^{-4}$
Variance of deviations due to the stratification of VAC by electrode diameter, S^2_{inad}	$5.8568 \cdot 10^{-3}$	$1.0449 \cdot 10^{-1}$
Fisher number of stratification of VAC by electrode diameters, F_{inad}	8.5	154
Tabulated values of Fisher number, $F_{tab.}$ (5%/1%)	2.25/3.12	2.51/3.62

formula with this argument experiences a substantial spread in diameter indicates the necessity of increasing the number of generalized arguments, in which the electrode diameter should be of considerable importance. The number $Gd\sigma_0h_0/I^2$ accounts for the influence of convective heat transfer on the generalized VAC. It is known that this process is predominant in the arcs of vortex plasmatrons. However, as the calculation shows, some other processes, have a pronounced effect on the VAC.

It is conceivable that only the allowed-for process of convective heat transfer is important inside the arc column. But the arc voltage is dependent not only on it but also on arc column dimensions, which are governed by the gasdynamics of flow in the zone of the "cold" gas blowing the arc. The dependence of arc dimensions on gasdynamic factors is thus far poorly investigated. It is conceivable that for arcs with a fixed length we can obtain sufficiently exact formulas without allowance for gas dynamic processes when the corresponding size of the arc column is used in place of the electrode diameter. Unfortunately, the column diameter is not an independent variable: it changes under the effect of other factors (current, gas flow rate, diameter of discharge channel). Therefore, in generalized VACs, we must use the corresponding electrode size instead of the diameter of the arc column.

To describe the characteristics of a plasmatron, we can assume that gas-dynamic phenomena in the "cold" zone depend on energy-transfer processes in the arc column itself. In this case, to reduce the electrode-diameter spread of the generalized VAC, we should take into account other energy-transfer processes in addition to convective heat transfer. Analysis shows that, for atmospheric-pressure arcs, energy transfer by conduction and turbulent transfer are the most significant. Generalized VACs that use more than one argument have a substantially smaller electrode diameter spread. For example, use of the criterion of turbulent energy transfer in addition to convective transfer yields $F_{add} = 48$.

Conclusions. The statistical analysis of the experimental data shows that a change in the diameter of the tubular electrodes of a vortex plasmatron has a certain effect on the voltage of an arc with self-adjusting length. However, the effect of a change in diameter is rather slight, and, in a first approximation, we can disregard it when approximating a VAC by a simple power expression. In generalized VACs with one predominant criterion of convective heat transfer, stritification by electrode-diameter is increased appreciables, which indicates the need to use a large number of generalized arguments. The reliable choice of a second criterion is currently made difficult because of insufficient data on the interaction of arc discharges with blowing flows.

NOTATION

d, electrode diameter; G, flow rate of air; F, ratio of Fisher variances; I, current; N, number of the degrees of freedom; S^2, experimental value of variance; SS, sum of the squares of deviations; t, Student quantile; U, arc voltage; π, criterion; c, constant; α, significance level for Fisher distribution. Subscripts: dev, deviation; add, additional; conv, convective; inad, inadequacy; n.err, net error; regr, regression; tab, tabulated; 0, scale value.

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